Submitted to Applied Physics B. Jan. 24, 1995

Mirror Birefringence in a Fabry-Perot Cavity and the Detection of Vacuum Birefringence in a Magnetic Field

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Abstract. We discuss the effect of mirror birefringence in two optical schemes designed to detect the quantum-electrodynamics (QED) predictions of vacuum birefringence under the influence of a strong magnetic field, ***B***. Both schemes make use of a high finesse Fabry-Perot cavity (F-P) to increase the average path length of the light in the magnetic field. The first scheme, we called the frequency scheme, is based on measurement of the beat frequency of two orthogonal polarized laser beams in the cavity. We show that mirror birefringence contributes to the detection uncertainties in first order, resulting in a high susceptibility to small thermal disturbances. We estimate that an unreasonably high thermal stability of - 10^{-9}K is required to resolve the effect to 0.1 %. In the second scheme, which we called the polarization rotation scheme, laser polarized at 45° relative to the ***B*** field is injected into the cavity. The ellipticity and polarization rotation of the light exiting the cavity is measured [1]. Under this scheme, mirror birefringence contributes as a manageable correction of the QED effect. We estimate that a thermal stability of -1 mK is sufficient.

PACS: 42.60Da, 42.60 Kg, 12.20Fv

The present work is motivated by the discontinuation of the Superconducting Super Collider (SSC) project, making available many magnets and cryogenic facilities to conduct research relating the retardation and absorption of light in a high magnetic field. The possibility of this type of experiment was first discussed by Iacopini and Zavattini [1]. Ni et al [2] have also proposed a similar experiment using a Michelson type interferometer. The first experiment was attempted by Cameron et al. [3], using the optical scheme of Ref. [1]. Their sensitivity was snort of that required to observe the QED effect by a factor of 100. Wc refer the readers to Ref. [3] for an excellent description of the experiment and theory. Here we shall present just enough to lead the reader to the problems associated with the birefringence of the mirrors.

The experiment involves sending a laser beam perpendicular to the direction of an applied magnetic field, **B**, and looking for the dependence of the speed of light on the polarization. The QED predicts that light with polarization parallel to the B field is retarded more than light with polarization perpendicular to the B field. In particle physics, the postulated existence of a pseudo-scalar particle known as the axion, is predicted to have a similar effect of much smaller magnitude. The theoretical predictions and the magnitudes of the effects are summarized below:

For QED, the difference in the refractive index is:

$$n_{//} - n_{\perp} = \frac{2\alpha^2 B^2}{\text{151?1:}} \qquad (1)$$

where $n_{//}$ and $n_{,}$ are the refractive indices of light for polarization parallel and perpendicular to the magnetic field, $\alpha = e^2/4\pi$ is the fine structure constant, m_e is the electron rest mass. The Lorentz-Heavyside units are used , where $h = c = 1$, $\alpha = 1/1\ 37$; 1 T of field and 1 m of length are equivalent to 195 CV^2 and $5.07\times10^6\ eV^{-1}$ respectively.

For 6T of field available with the SSC magnet, $n_{//} - n_{\perp} = 1.43 \times 10\text{-}2$). If light polarized at 45° relative to $\boldsymbol{B}$ is reflected an average of N times in an ideal mirror with no intrinsic birefringence, it will acquire an ellipticity,

$$\psi_{QED} = \pi NL(n_{//} - n_{\perp}) / \lambda, \tag{2}$$

where L is the length of the high field region, λ is the wave length of the laser light. The ellipticity is half the phase difference between the orthogonal components. In state-of-the-art mirrors N = 10^6 is achievable. The wave length for Nd:YAG laser is 1.064 µm. The clear aperture of the SSC magnet bore is 38 mm. We have optimized the magnet string length against diffractive loss of the laser [4]. We found that for this aperture the optimal $\boldsymbol{L}$ is 120m. With these values, $\psi_{QED} = 5 \times 10^{-8}$ rad. In addition, QED predicts that there is no absorption of the light.

For axion detection, the predicted ellipticity is:

$$\psi_a = \frac{N g_{a\gamma\gamma}^2 B^2 \omega^2}{2 m_a^4}\left[\frac{m_a^2 L}{2\omega} - \sin\left(\frac{m_a^2 L}{2\omega}\right)\right], \tag{3}$$

where m_a is the mass of the axion, $g_{a\gamma\gamma}$ is the axion-two-photon coupling strength. In addition, the polarization angle is predicted to rotate by:

$$\varepsilon_a = \frac{N g_{a\gamma\gamma}^2 B^2 \omega^2}{m_a^4} \sin^2\left(\frac{m_a^2 L}{4\omega}\right). \tag{4}$$

This rotation is caused by a small absorption of the light with polarization parallel to B. Axions with mass greater than 10-3 CV have been ruled out either by experiments or by astrophysical observations. For mass less than 10^{-3} eV, the effect is at least 10^9 times smaller than the QED effect; i.e. $\psi_a \approx \varepsilon_a <$ 10-16 rad.

1. Mirror Birefringence

The mirrors used in high finesse Fabry-Perot cavities arc dielectric mirrors. These mirrors and their coating materials are not intrinsically birefringent. Mirror birefringence is caused by stress in the dielectric coating material. The stress results from the difference in the thermal expansion coefficients between the coating materials and the substrate material. The best dielectric mirrors reported in the literature [5] have a birefringence, ϕ_m, between 1 and 10 μrads per bounce. This means that after one reflection, linearly polarized light along the "fast" axis of the mirror will be phase shifted by ϕ_m relative to light polarized along the "slow" axis. This mirror birefringence can vary by about 1 % from spot to spot over the mirror surface, When light is reflected 10^6 times in a F-P, the cumulative phase difference can be as large as -1 rad. Linear polarized light entering the cavity at 45° to the "fast" axis will emerge as almost circular polarized. To use these mirrors meaningfully for the purpose of detecting vacuum birefringence, the polarization of the laser in the cavity must be aligned along either the "fast" or the "slow" axis of the mirrors. In this case, the cavity resonant frequencies for the two orthogonal polarization modes will differ significantly. We calculated the difference, Δf_m. and compared it to the resonant width of the F- P to get an estimate of the effect of mirror birefringence on the finesse of the cavity if the laser polarization is misaligned with the "fast" (or "slow") axis by a small angle θ_o. We found that $\Delta f_m = C\phi_m / 4\pi L_c$, where L_c *is* the length of the cavity. For a 150m long cavity and ϕ_m of 10 μrad per bounce, $\Delta f_m = 1.6$ Hz. 'l'he free spectral range for this cavity is 1 MHz. For N of 10^6, the full width at half maximum of the cavity fringe is $\Delta f_{1/2} = 0.64$ Hz. Since Δf_m ***and*** $\Delta f_{1/2}$ are of the same order of magnitude, the cavity resonance is not affected much by a small misalignment angle θ_o. However as we shall show later, the polarization rotation scheme is very sensitive to θ_o.

2. Frequency Scheme

Figure 1 shows that the polarization of the two laser beams in the frequency scheme arc orthogonal and arc aligned with the "fast" and "slow" axis of the mirror. One of these polarization, is also parallel to $\boldsymbol{B}$. The difference in the resonance frequencies of the two orthogonal modes is:

$$f_{\perp} - f_{//} = \frac{c}{2L_c}\left(\frac{L_c\left(n_{//} - n_{\perp}\right)}{\lambda} + \frac{\phi_m}{2\pi}\right), \tag{5}$$

Putting in the values mentioned above, it can be shown that mirror birefringence contributes a frequency difference of 1.6 Hz, while the QED effect contributes a mere 2x 10-81 Iz. To resolve the QED effect to 0.170 would require the mirror birefringence to bc stable to one part in 10^{11}, during the 10 minute period of a magnet ramp. A major concern is the variation of the mirror birefringence with temperature. Although the temperature coefficient has not yet been measured, it can be reasonably estimated as follows: Mirrors arc coated at 200° C, the stresses are induced when the coatings arc subsequently cooled to room temperature. Therefore, if 10 μrad pcr bounce is measured at room temperature the temperature coefficient is estimated to be 10 μrad/200°C, or ***0.05*** μrad /°C pcr bounce. To have onc part in 10^{11} stability, the temperature must be stable to 2x10-9 "C. Although temperature measurement and control of better that 10^{-9} K has been demonstated [6], they arc achieved with cryogenic technology and require careful shielding from heat radiation. Such technology is not yet available at room temperature. With the required kWatts of power in the cavity to obtain low enough photon noise for the measurement, the task required to stabilize the temperature of the mirrors is daunting. The main problem with the frequency scheme is that mirror birefringence contributes to the uncertainties in first order. In the rotation scheme discussed below, wc shall show that the mirror birefringence contributes as a correction of the small QED term.

Figure 1 to be inserted here

3. **Polarization Rotation Scheme**

This scheme is a variant of the scheme proposed by Iacopini and Zavattini [1]. A laser polarized at 45° with respect to the ***B*** field is injected into the F-P. The mirror's "fast" or "slow" axis must be aligned with the direction of the polarization, as shown in Figure 2. In this configuration the linear polarization state of the laser is preserved. When ***B*** = O, any changes of the mirror birefringence dots not alter this polarization state except if the principle axis of the mirror rotates with temperature. Such a possibility is second order and will be discuss later. When a strong ***B*** field is present in the cavity, the vacuum birefringence causes the polarization state to be converted to slightly elliptical. However, the light exiting the cavity is a mixture of rotated light and elliptic light because of the birefringence of the mirror, which partially converts the elliptic light to rotated light. The experimental task is therefore to measure the elliptic component, the rotated component and the mirror birefringence, and mathematically compute the vacuum birefringence from these measurements. The mathematical relations are formulated below.

Figure 2 to be inserted here

Let ***i*** and ***j*** be unit vectors perpendicular and parallel to ***B*** respectively; and ***i'***,***j'*** be unit vectors rotated by 45°. Let the ellipticity after N bounces in an ideal mirror with no birefringence be Ψ'. Thus $\Psi' = \Psi'_{QED} + \Psi'_a \approx \Psi'_{QED}$. We construct a 45° standing wave inside the cavity and shift the phase of the i component forward and the j component backward by Ψ'.

$$E = A\, F(r) \sin(\omega t - \Psi')\, i + D\, F(r) \sin(\omega t + \Psi')\, j, \tag{6}$$

where the amplitude A and D arc matched as closely as possible to ensure good alignment at 45°; $F(r)$ describes the sinusoidal spatial variation. It remains constant throughout the analysis. The small misalignment angle θ can bc related to A and D by:

$$Tan\left(\frac{\imath\pi}{\imath 4} + \theta\right) = \frac{D}{A} \quad \text{or} \quad 2\theta = \frac{A - D}{D} \text{“} \tag{7}$$

The θ can bc decomposed as the sum of a DC misalignment term, θ_0, a term that changes with temperature, $\beta(T - To)$, representing second order mirror birefringent effect, and axion polarization rotation, ε_a, which varies with **B**, i.e. $\theta = \theta_0$ -t $\beta(T - T_o) + \varepsilon_a$, where T_o ***is*** an arbitrary initial temperature. In terms of θ,

$$E = D\, F(r) [\sin(\omega t - \Psi')\, i \text{ -t } \sin(\omega t + \Psi') j + 2\theta \sin(\omega t - \Psi')\, i\,]$$

$$= D\, F(r) [\sqrt{2}(\cos\Psi' - \sin Y') \sin \omega t\; j' \text{ -<2 } \sin \Psi' (\cos\omega t\; i' - \sin \omega t\; j')$$

$$+ \sqrt{2}\,\theta \sin(\omega t - Y')\, (i' + j')\;]. \tag{8}$$

In the expression above we have decomposed the polarization state of the light into liniar polarized components and circular polarized components. The first term is the original laser polarized at 45°, The second term is the circular polarized light resulting from vacuum birefringence. The third term is caused by misalignment and axion rotation. Mirror birefringence is included into the calculation by shifting the phase along i' by Ψ'_m and along j' by - Ψ'_m, where $\Psi'_m = N\phi_m/2$.

$$E = 1> F(r) \{ \sqrt{2}(\cos\Psi' - \sin\Psi') \sin(\omega t - \Psi'_m) j'$$

$$-42 \sin Y'[\cos(\omega t + \Psi'_m) i' - \sin(\omega t - \Psi'_m) j']$$

$$+ \sqrt{2}\,\theta [\sin(\omega t - \Psi' + \Psi'_m) i' + \sin(\omega t - \Psi' - \Psi'_m) j'] \}. \qquad (9)$$

Second term

$$= -2\sqrt{2} \sin Y' \mathit{fin}\, \Psi'_m \cos(\omega t + \pi/4) i - \sqrt{2} \sin\Psi' (\cos\Psi'_m - \sin\Psi'_m) [cm\, \omega t\, i' - \sin \omega t\, j'] . \qquad (lo)$$

Third term

$$= 20\ (\cos\Psi'_m + \sin\Psi'_m) \sin(\omega t - \Psi') i + 2\theta \sin\Psi_m [\cos(\omega t - \Psi') i + \sin(\omega t - \Psi') j] . \qquad (11)$$

From these relations, it can easily bc seen that mirror birefringence only contributes as a correction to small quantities of order Y' and θ. The dominant magnetic field dependent term is the second term. The third term depends on the magnetic field weakly through ε_a, which is at least 10^9 times smaller. However, if θ_0 is large enough, the temperature variation of Ψ'_m can contribute as a significant noise source. To estimate the required alignment accuracy, wc note that the second term must bc known to 5x 10-11 to obtain an accuracy of O. 1 % in Y' (Y' $\approx 5\times10^{-8}$ rad). If the ten iperature of the mirror is stable to -1 mK, Ψ'_m is stable to -2x 10^{-5} rad.Since the third term is of order $\theta\Psi'_m$, θ must bc no more than -10-6 rad. Alignment to such high precision is most easily achieved by injecting a small DC current through the coil of a Faraday cell. The orientation of the mirror's principle axis must be stable to within 10^{-6} rad against temperature changes. This imposes a requirement on the thermal rotation coefficient β. For 1mK temperature stability, β must bc less than 1 mrad/K. This value is not unreasonable for, mirror. However, the actual value must be experimentally verified to bc within this specification.

Figure 3 shows the experimental scheme to detect the vacuum birefringence. The polarization rotation of the light from the reflected end of the cavity is measured simultaneously while measuring the ellipticity from the transmitted end of the cavity. We refer the readers to Ref. [1] and [3] for a detailed description of the techniques for measuring polarization rotation and ellipticity. It is sufficient to note here that the technique requires modulating the laser polarization angle by 10^{-3} rad at a frequency ω_1 of a few hundred Hertz. However, in the presence of mirror birefringence, it is not desirable to modulate the polarization angle of the light inside the cavity by an amplitude larger than 10^{-6} rad. Doing so is equivalent to introducing an additional modulated term of - 10^{-3} rad in θ, resulting in many terms with the modulated frequency and twice the modulated frequency, as can be seen by computing (second term + third term)^2. This will also couple in undesirable thermal effects. Another reason is that the storage time τ of the cavity is about 0.5 sec. The output light of the cavity will be an average of the light that entered the cavity in the last 0.5 sec. Thus the cavity will behave like a low pass filter with a time constant τ. Any signal modulated at frequencies higher than $1/(2\pi\tau)$ will be severely attenuated. Therefore the Faraday modulators for the detection of ellipticity and polarization rotation must only act on the light exiting the cavity,

Figure 3 to be inserted here

A second low power laser orthogonally polarized to the main laser (see Figure **2)** is activated periodically to measure the mirror birefringence using the beat frequency method. Because Ψ'_m only needs to be known to O. 1 % and is intrinsically stable to 0.001 % for 1 mK temperature variation, we do not expect that real-time measurement of Ψ'_m is necessary.

The scheme presented above is sufficient to measure Ψ'_{QED}. The more challenging task is to develop a scheme to detect axions. Axion detection can be accomplished by comparing the measured ellipticity with the prediction of QED. Any deviation over the uncertainty of the measurement is a possible axion signal. This method has a limitted accuracy because the comparison with the theory also requires high accuracy measurements of the magnetic field, the finesse of the cavity, and the length of the magnetic region. To carry out measurements of these quantities to better than 0.1 % will be difficult. At 0.1 %, higher order QED effects will become important and need to be carefully computed. In the absence of mirror birefringence, a more straightforward method is to measure the polarization rotation. Because the QED effect does not produce a polarization rotation, any observed rotation can be attributable to axion. But because of mirror birefringence, the QED ellipticity and the axion polarization rotation effects are mixed. Fortunately, the effect of mirror birefringence can be easily compensated by passing the light exiting the cavity through a thin slice of a birefringent material for coarse correction and a Kerr cell for fine confection. The thickness of the slice is chosen so that more than 99% of the mirror birefringence is corrected. The Kerr cell, which is a device to introduce an adjsutable birefringence, is used to compensate the rest of the birefringence. This arrangement is shown in Figure 4 below. Two such cells arc inserted into the beams exiting the cavity at points labeled C in Fig. 3. By applying a correction DC voltage across the cells, light with polarization along the "fast" axis of the mirror is slowed by just the right amount to cancel all birefringence effects of the mirror. The principle axis of the birefringent slice and that of the Kerr cell must be carefully aligned with the mirror's principle axis. The voltage applied across the Kerr cell must bc stable and controllable with high precision.

Figure 4 to bc inserted here

The fine-tune adjustment of the Kerr cell is best performed by inserting either a Faraday modulator or a Kerr modulator into the light beam entering the cavity, i.e. at point A of Fig. 3. The Faraday modulator allows introducing an AC rotation of -0.1

Hertz into the cavity (this frequency must be less than $1/(2\pi\tau)$). When it is replaced by a Kerr modulator, an AC ellipticity is injected instead. The amplitude should be about 10 to 100 times times larger than the QED effect (or ~ 10^{-6} rad). For polarization rotation detection, the DC voltages of the Kerr cell is adjusted so that the detector would not show any signal if an elliptic component is injected into the cavity. For ellipticity measurement, the Kerr cell voltage is adjusted so that the detector would not show any signal when a modulated rotation signal is injected into the cavity. These adjustments are performed at zero magnetic field. It should be possible to compensate mirror birefringence to 0.001 %, where the stability of the birefringence becomes the limiting factor.

This modified scheme not only allows separating the axion rotation signal from the QED ellipticity signal, it also allows another way to measure the QED ellipticity more directly.

Because of the non-uniformity of the mirror birefringence, it is important to keep the relative position of the beam fixed with respect to the mirror. We estimate the requirement on the position stability by assuming the birefringence to be 10 μrad/bounce and the change in the birefringence over a distance of 1 cm to be ~1%. For the frequency scheme the relative position must be stable to 10-12 m to meet the 0.1 % QED effect requirement. For the polarization rotation scheme, this requirement is relaxed by a factor of 10^6.

5. Conclusion

We crudely estimated the thermal effect of mirror birefringence based on a model of thermally induced stress in the mirror coating. We assumed that at the substrate deposition temperature of 200° C, the mirror birefringence is zero. This estimation may be off because the coating material condenses from vapor, which is likely to be at a much higher temperature. However, even if the temperature at which the birefringence is zero were 2000° C instead of 200° C, our estimation is only off by a factor of ten, which would not affect the main conclusions. Real measurement of the temperature coefficient of mirror birefringence is urgently needed. When measured values are available, the estimates must be updated. But in comparison with the frequency scheme, the polarization rotation scheme is always about 10^6 times less sensitive to any changes in the mirror birefringence. This result is independent of the cause of the changes or the magnitude of the changes.

Acknowledgment: This work was supported by the Department of Energy under Interagency Agreement #IAA DE-A105-94ER4O8S7.

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FIGURE CAPTIONS

Figure 1: The alignment of the lasers in the frequency measurement scheme.

Figure 2: The alignment of the lasers in the rotation measurement scheme.

Figure 3: The optical scheme of the experiment to measure the ellipticity and polarization rotation simultaneously.

Figure 4: Scheme to correct mirror birefringence. This arrangement is to be inserted into the beams exiting the cavity at points labeled C in Figure 3.

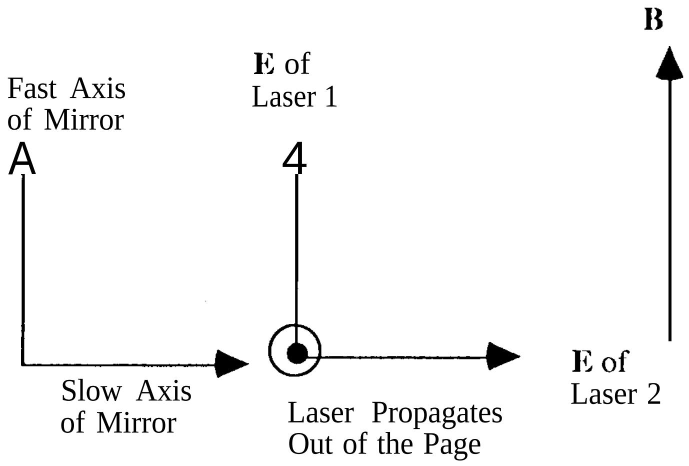
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Figure 1:

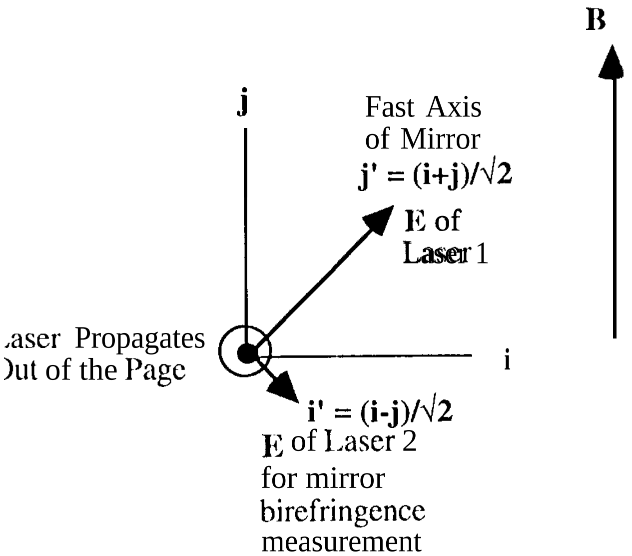
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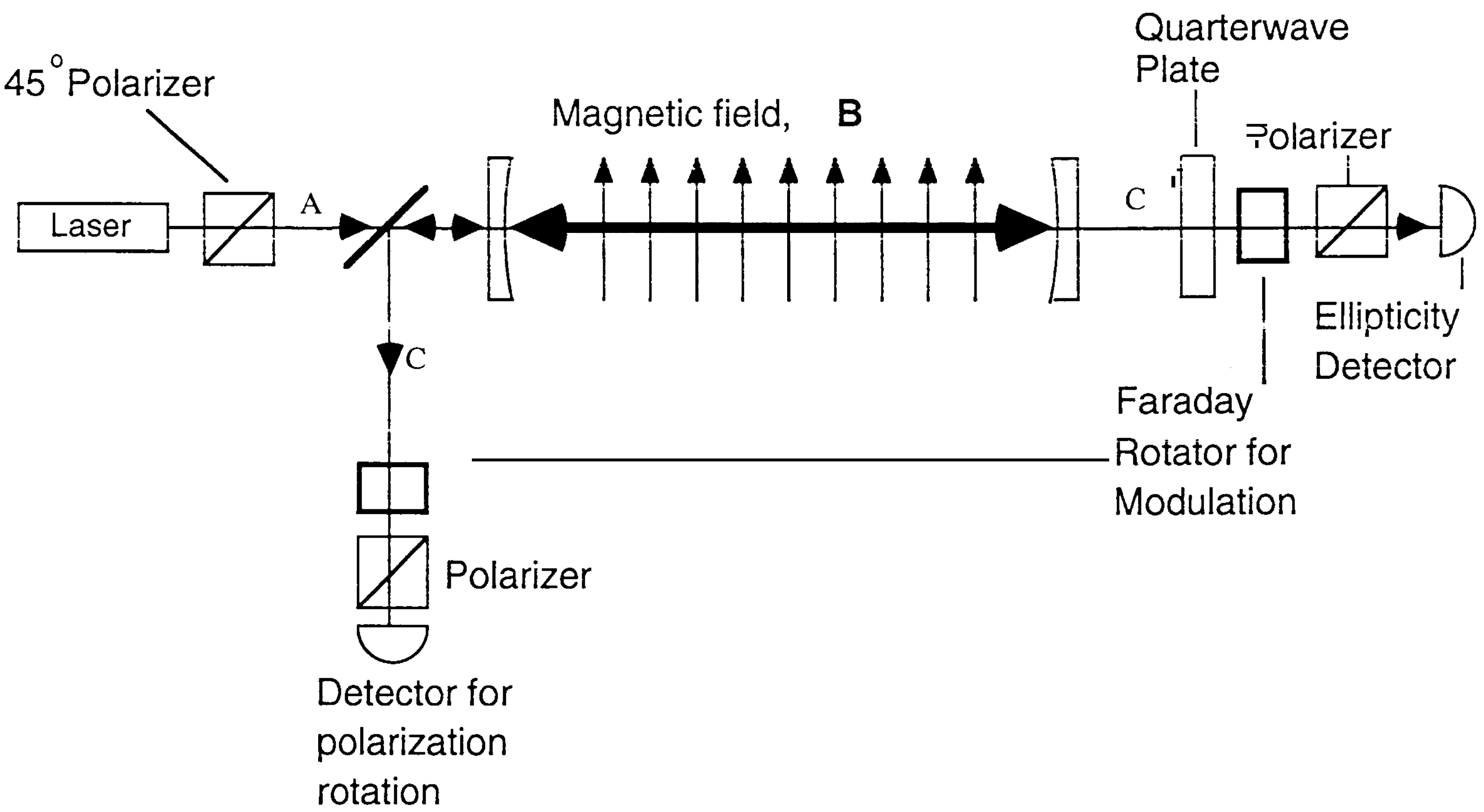
Figure 2:



45° Polarizer
Laser
A
Magnetic field, B
Quarterwave
Plate
Polarizer
C
Ellipticity
Detector
Faraday
Rotator for
Modulation
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Polarizer
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polarization
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Fig. 3

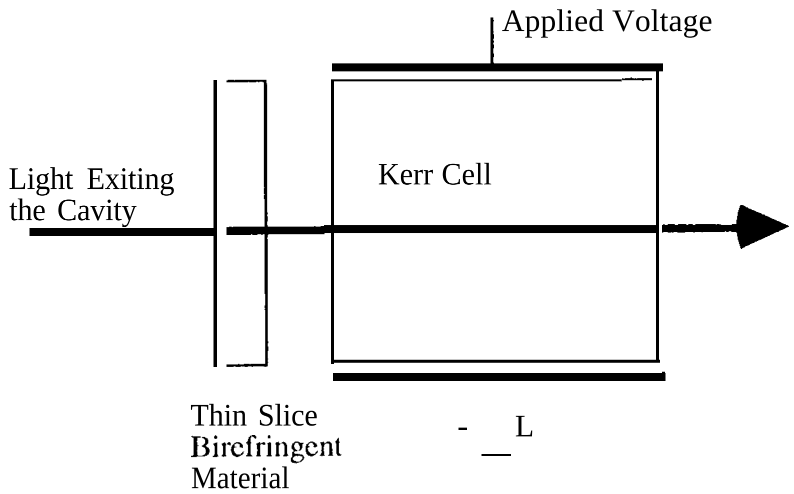
Applied Voltage
Kerr Cell
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Figure 4: